1. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

a) Arithmetic Sequence : $a_n = a + (n-1)d$

a_n = n'th term
a = first / initial term ($28\ kmh^{-1}$)
d = common difference between terms.

$a = 28$, $a_6 = 115$

$\Rightarrow a_6 = 115 = 28 + (6-1)\cdot d$

$\Rightarrow 5d = 115 - 28 \Rightarrow d = \frac{115-28}{5} = 17.4$ (1)

$\Rightarrow a_3 = 28 + (3-1)\,17.4$ (1)

$\Rightarrow a_3 = 62.8\ kmh^{-1}$ is the fastest speed of the car in 3rd gear. (1)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

b) Geometric Sequence : $a_n = ar^{n-1}$

a_n = n'th term
a = first / initial term
r = common ratio between terms

$a_6 = 115\ kmh^{-1}$ and $a = 28\ kmh^{-1}$

$\Rightarrow a_6 = 115 = 28 \cdot r^5$

$\Rightarrow r^5 = \frac{115}{28} \Rightarrow r = \left(\frac{115}{28}\right)^{1/5} = 1.3265\ldots$ (1)

$\Rightarrow a_5 = 28 \cdot (1.3265\ldots)^4 = 86.6941\ldots$ (1) $\Rightarrow a_5 = 86.7\ kmh^{-1}$ is the fastest speed of the car in 5th gear. (1)

2. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

a) $a_2 = a_1 + (n-1)d$ ← formula. $a_n = a_1 + (n-1)d$

$24 = 16 + (21-1) \times d$ (1)

$24 = 16 + 20d$

$8 = 20d$

$0.4 = d$ (1)

b) $S_n = \frac{1}{2} n [2a + (n-1)d]$

$S_{500} = \frac{1}{2} \times 500 [2 \times 16 + (500-1) \times 0.4]$ (1)

$S_{500} = 250 (32 + 199.6)$

$S_{500} = 250 \times 231.6$

$S_{500} = 57900$ (1)

OR use $S_n = \frac{1}{2} n (a+l)$

$l = 16 + (500-1) \times 0.4$

$l = 215.6$

$S_{500} = \frac{1}{2} \times 500 (16 + 215.6)$

$S_{500} = 57900$